# Causal-Convolution—A New Method for the Transient Analysis of Linear Systems at Microwave Frequencies

Thomas J. Brazil, *Member, IEEE*

***Abstract*—A new convolution-type method is presented for the transient analysis of causal linear systems described in the frequency-domain. The central novelty lies in the proposed method of determining impulse responses in the time-domain, which are interpreted as truly discrete functions corresponding to periodically-extended system functions in the frequency-domain. Such impulse responses may be computed with high numerical efficiency, while having excellent interpolation properties with respect to the original system function. The convolution operations which result are also naturally in the form of a sum-of-products calculation. The method is capable of handling arbitrary excitation signals, and may in principle be readily extended to more general nonlinear analysis. Several examples of the technique are given, including comparisons and validation both using existing methods, analytical results and experimental measurements.**

## I. Introduction

EFFICIENT and flexible large-signal analysis capability is a critical requirement for many types of microwave CAD. Because of the complexity of the circuit and device models involved in such simulations, considerable challenges arise in finding a suitable general-purpose method of numerical solution. This contribution describes a new approach to the characterisation of causal linear systems which provides a very attractive basis for analysing the more general nonlinear problem. However, in order properly to describe the basis and range of applicability of the proposed approach, we restrict our attention in the present paper solely to the transient analysis of linear, time-invariant electrical systems, subject to essentially-arbitrary excitation signals. The method has already been described in brief outline in [1]: in the following, a more extensive discussion of the mathematical basis is provided, and important issues affecting accuracy and areas of applicability are addressed.

The literature on the transient numerical solution of linear systems is, of course, very extensive, and in this introduction we consider only briefly those methods more commonly encountered by microwave engineers, to whom two basic situations usually present themselves. In the first case, the composition of the linear system is known, and a time-domain description is possible in terms of a finite system of ordinary differential equations (ODE's), such as in the familiar case of lumped circuit elements. Numerical solution of the ODE's involves their transformation to difference equations, and it is possible within this context to extend the analysis to include distributed circuits consisting of ideal transmission lines. The SPICE program is probably the most well-known electrical analysis tool for this kind of problem, but many others exist. A reasonably wide range of common excitation signals are accommodated within SPICE, and transient analysis proceeds by standard step-by-step, time-domain solution techniques.

A second, not necessarily separate, situation arises when the terminal behaviour of the linear network is specified in terms of a complex-valued system function in the frequency-domain. This may be achieved either by direct calculation for a known network structure, by measurement, or through some other process. A very common numerical analysis approach in such cases involves sampling the excitation signal in the time-domain, followed by application of the Discrete Fourier Transform (DFT) to produce a representation in the frequency-domain—an operation which may benefit greatly from the computational efficiency of the Fast Fourier Transform (FFT) algorithm [2]. The solution is then carried out in terms of frequency-domain samples, and an inverse DFT is carried out to recover the time-domain solution [3]. Indeed, this procedure is theoretically exact, at least for the steady-state response, provided the excitation signal is band-limited and sampled at least at the Nyquist rate. If these conditions are *not* satisfied, however, aliasing errors occur which are difficult to estimate and/or may require large transform sizes for their minimisation. It may be noted that the extension of this latter technique to mixed linear/non-linear system solution leads to the well known Harmonic-Balance method in its various forms [4], [5].

The method proposed in this work is based on a convolution technique, with the main novelty residing in the method of obtaining and using the impulse response samples. Convolution-oriented techniques have been proposed by a number of authors (e.g. [6]–[8]), but they either continue to rely on direct application of the DFT to interconnect time- and frequency-domains, or are restricted in the class of linear problem which may be addressed (e.g. nearly-matched transmission lines). Of course, in mathematical terms, the transient response is also

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The author is with the Department of Electronic & Electrical Engineering, University College Dublin, Dublin 4, Ireland.
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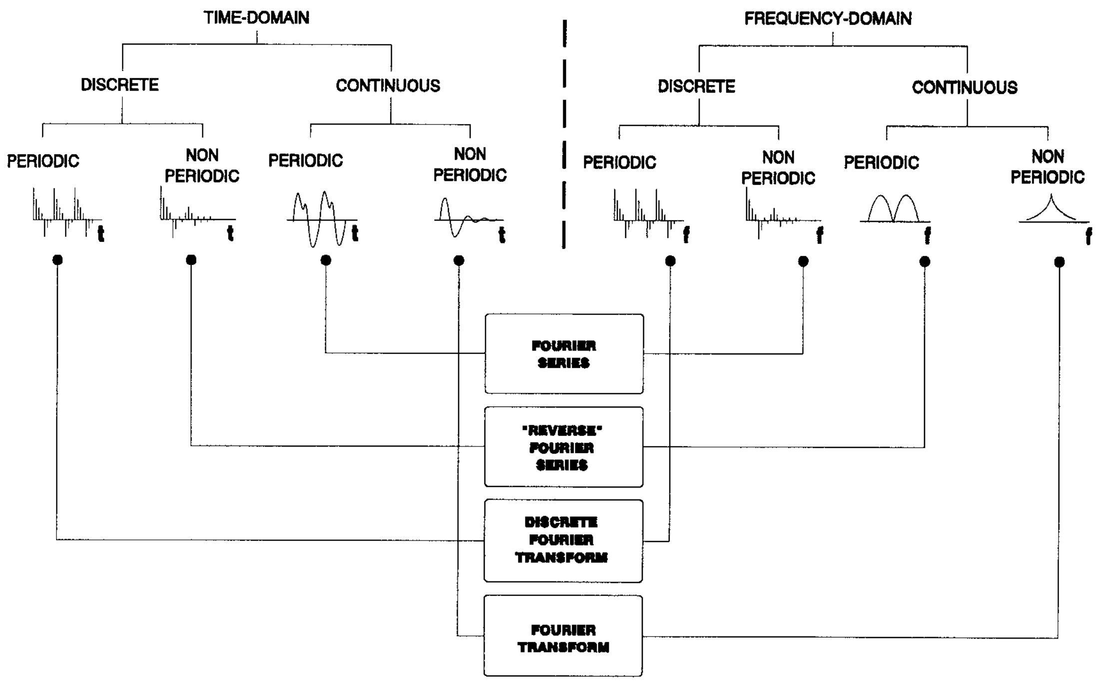


Fig. 1. Classification of time-domain and frequency-domain relationships.

obtainable from the inverse Laplace Transform, and, while numerical implementation of this approach is possible [9], considerable care is needed in practice to produce reliable results. A range of other approaches to transient analysis have been proposed, including different methods based, for example, on "waveform relaxation" or "asymptotic waveform expansion (AWE)," which are applicable to lossy transmission line problems [10], [11].

In Section II we present the linear-system and signal-processing background to the method proposed here, described as the Causal-Convolution method. This is followed in Sections III and IV by an account of the practical implementation issues that must be considered for applying this method. A selection of representative results is contained in Section V, aimed at assessing and validating the technique in comparison both with other methods, and with experimental measurements. A discussion and conclusions are contained in Section VI.

## II. Linear-System and Signal-Processing Basis of the Causal-Convolution Method

The essential framework for the method described here may be introduced with reference to Fig. 1. This diagram classifies functions which are described equivalently in the time-domain and (real) frequency-domain, into the sub-categories of discrete and continuous, in the sense of the range of the independent variable. These are further divided within each class as either periodic or non-periodic. For present purposes, all frequency-domain representations are assumed to exhibit Hermitean symmetry, i.e.:

$$X(f) = X^*(-f) \tag{1}$$

where "*" denotes complex conjugate, and the time-domain functions are accordingly real-valued. It is assumed further that all time functions have finite average power, whether averaged over one period (in the periodic case) or over all time. Note that the representations of Fig. 1 may be taken to apply either to signals or systems. For example, in the case of signals, the frequency-domain representation would be interpreted as the *spectrum* of the signal, whereas for systems, the time-domain function constitutes the *impulse response* of the system, and so on.

The Fourier Transform (FT) provides a general relationship between the two domains, and in fact, using the mathematical theory of distributions, all four transform-pair relationships indicated in Fig. 1 may be viewed as particular cases of the FT. However, it is considered clearer in the present context to reserve use of the FT for continuous, non-periodic functions in time and frequency. Of the remaining three cases, the DFT is observed to be appropriate to functions which are inherently periodic in both domains—this gives rise to the aliasing problems mentioned earlier when it is used with non-band-limited signals. Moreover, the DFT is strictly defined for *discrete* functions only—extrapolation to continuous behaviour requires great care. In the design of Finite Impulse Response (FIR) digital filters, for example, it is well known that direct use of the DFT produces poor interpolation behaviour between sample points in the continuous frequency domain [12].

A further transform pair in Fig. 1 is the familiar form of the Fourier Series, while the fourth situation is of most direct interest in what follows. This transform relates continuous, periodic, complex-valued functions in the frequency-domain, to discrete, real-valued functions in the time-domain. A straightforward approach to such cases would be to use a reverse form of Fourier Series, as follows, where $2.\omega_m$ is taken as the period of the frequency-domain function:

$$\begin{aligned} x(nT) &= \frac{T}{2\pi} \cdot \int_{-\omega_m}^{+\omega_m} X(\omega) \cdot \exp[+jn\omega T] \cdot d\omega \\ X(\omega) &= \sum_{n=-\infty}^{+\infty} x(nT) \cdot \exp[-jn\omega T] \\ &= x(0) + \sum_{k=1}^{+\infty}[x(kT) + x(-kT)] \cdot \cos(k\omega T) \\ &\quad - j\sum_{k=1}^{+\infty}[x(kT) - x(-kt)] \cdot \sin(k\omega T) \end{aligned} \tag{2}$$

where $T = (\pi/\omega_m)$.

Notice the exponent sign-change compared to the usual complex form of the Fourier Series, the reason for which is explained in due course. Using (1) it is easy to confirm that the above integral for the time-domain samples gives a purely real-valued result, which, with $X(\omega) = R(\omega) + j \cdot I(\omega)$, may be evaluated as:

$$x(nT) = \frac{1}{\omega_m} \cdot \int_0^{\omega_m} [R(\omega) \cdot \cos(n\pi T) - I(\omega) \cdot \sin(n\pi T)] \cdot d\omega \tag{3}$$

Assuming the periodic frequency-domain function in question to be a system function, then (2) shows that the associated time function, which is a form of impulse response, is a discrete, real-valued function extending over positive and negative time. The approach just described is quite general, in that the real and imaginary parts of the system function may be defined independently, provided (1) is satisfied. However, all physically-realisable passive systems must exhibit the property of *causality*, meaning simply that if the input is zero for time $t < t_0$, the output is also zero for $t < t_0$. Hence, the impulse response must also be zero for negative time. In terms of the associated system function, this places quite severe additional constraints on the frequency-dependence of the real and imaginary parts (or amplitude and phase functions), besides the condition of (1), which merely ensures that the impulse response is real. For periodic functions, these conditions may be expressed as the following Hilbert Transform pair [2]:

$$\begin{aligned} I(\omega) &= -\frac{1}{2\pi} \cdot \int_{-\pi}^{+\pi} R(\lambda) \cdot \cot\left(\frac{\omega - \lambda}{2}\right) \cdot d\lambda \\ R(\omega) &= x(0) + \frac{1}{2\pi} \cdot \int_{-\pi}^{+\pi} I(\lambda) \cdot \cot\left(\frac{\omega - \lambda}{2}\right) \cdot d\lambda \end{aligned} \tag{4}$$

It may be noted further that if the system function is *minimum phase* (i.e. has no zeroes in the open right-hand $s$-plane), the real-part function is uniquely determined by the imaginary-part function, and vice versa. Using (4) in (3), we find that $x(nT)$ becomes identically zero for negative values of "$n$," as would be expected for a causal time-domain representation. (This would not be the case if the opposite sign had been used in the exponents of (2), since the time-domain samples would then become anti-causal, i.e. zero-valued for positive time).

Assuming a causal system function in the following discussion, then a substantial improvement becomes possible with regard to the numerical calculation of impulse response samples, if the causal condition is in effect forced by the transform relationships. Hence, the following transform pair is proposed here for such cases, as a replacement for (2):

$$\begin{aligned} x(nT) &= \frac{1}{2\pi} \cdot \int_{-\omega_m}^{+\omega_m} X(\omega) \cdot \exp[+jn\omega T] \cdot d\omega \\ X(\omega) &= T \cdot \sum_{n=0}^{+\infty} x(nT) \cdot \exp[-jn\omega T] \\ &= T \cdot x(0) + T \cdot \sum_{k=1}^{+\infty} x(kT) \cdot [\cos(k\omega T) \\ &\quad -j \cdot \sin(k\omega T)] \end{aligned} \tag{5}$$

The change in scaling factors has been introduced to allow the $x(nT)$ to limit to the conventional (continuous) impulse response as $\omega_m$ tends to infinity. For numerical evaluation of (5), the infinite summations shown must be truncated. If a finite number of terms are sufficient in the summation, then it is easy to show that the integral evaluation in (5) reduces *exactly* to a trapezoidal-rule type of summation. In particular, if $(N + 1)$ equally-spaced samples of the system function in the frequency range $[0, \omega_m]$ are used in (5), the impulse response produced is of length $(2.N)$ in positive time. Then it is possible to take full advantage of the FFT algorithm, and obtain results with excellent accuracy, provided sufficient samples are taken of the frequency domain function so that the discrete impulse response tends to reasonably small values when computed at the largest time points. As demonstrated in the examples to follow, the impulse response calculated in this way is found to have excellent interpolation properties in the frequency domain, in the sense of fitting the original system function well *between* the $(N + 1)$ frequency sample points used for its derivation.

With the system function represented by a finite impulse response, the computation of the transient response in effect reduces to a discrete convolution operation at each time step. However, the precise nature of this operation depends on the formulation of the system function, as discussed in more detail in the next section.

The essence of the technique presented here is to define a methodology whereby all causal linear system functions may be represented approximately by a finite impulse response, up to some specified boundary frequency. Beyond this frequency, the (periodic) function to which the impulse response interpolates and the original system function, may deviate quite considerably. Therefore the main error in performing transient analysis using the discrete impulse response description, resides in the degree to which the spectral energy in the excitation signal becomes relatively small beyond the boundary frequency. The situation is depicted schematically in Fig. 2, where an amplitude response is shown together with both its periodic extension and an assumed spectral distribution for the excitation signal. Errors arise in transient analysis

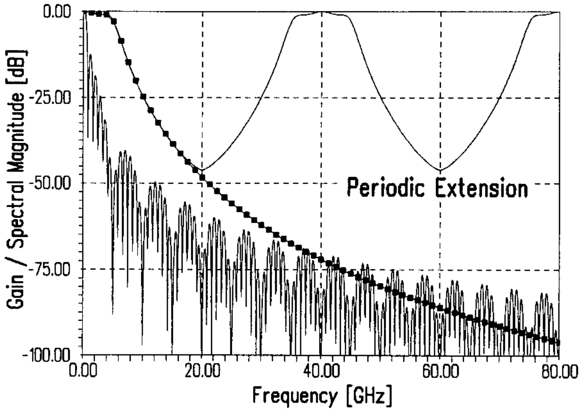


Fig. 2. Example of system amplitude function (■) and periodic extension, with spectral distribution of excitation signal ($f_m = 20Ghz$).

from that portion of the signal spectrum beyond $f_m$ which interacts with the periodic extension of the system function, rather than with the original system function. In practice, and as demonstrated in Section V, the technique is found to be rather tolerant of "leakage" of excitation signal spectral energy beyond the boundary frequency.

The overall process described above for computing the impulse response samples, combined with carrying out the appropriate convolution operations, will be described in the following as the *Causal-Convolution* (CC) technique for the transient analysis of linear systems.

## III. Naturally-Periodic System Functions

It is clear from the previous section that the CC-method is based fundamentally on the concept of a periodic system response. Of course, the majority of system functions encountered in practice do not directly satisfy this requirement, but as will be shown in due course, the CC-approach can still give excellent results in such cases provided certain preparatory steps are taken. For the present, many essential features of the technique may be illuminated by assuming that the system function is indeed periodic. A number of circuits familiar to microwave engineers perfectly fulfil this condition-indeed, any network composed of an arbitrary connection of ideal, commensurate transmission lines will suffice.

In order to focus the discussion, we consider the one-port network in Fig. 3. Let us suppose that the transient input current is required in response to a specified excitation voltage beginning at $t = 0$. A simple choice for the system function would be the input impedance $Z(\omega)$ which is assumed to be periodic in frequency over $[-\omega_m, \omega_m]$. If $(N+1)$ samples are

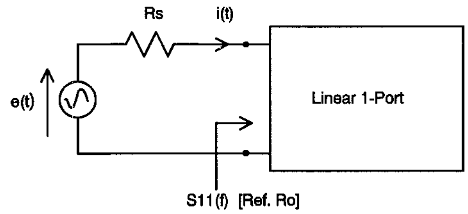


Fig. 3. Simple 1-Port network to demonstrate transient analysis technique.

taken in the positive frequency range, then impulse response samples $z(kT)$ may be computed as described in the previous section, with '$k$' in the range $\{0, (2N-1)\}$. Now in the frequency-domain we write:

$$E(\omega) = [R_s + Z(\omega)] \cdot I(\omega)$$

Using the CC-approach and transforming to the time-domain:

$$e(nT) = R_s \cdot i(nT) + T \cdot \sum_{k=0}^{2N-1} i((n-k)T) \cdot z(kT) \qquad (6)$$

Notice that instead of a convolution integral being required in the time domain, as would normally be the case, the discrete nature of $z(kT)$ means that the simple summation shown in (6) is *exactly* correct. The algorithm for computing the $i(nT)$ is therefore:

$$i(nT) = \frac{[e(nT) - T \cdot \sum_{k=1}^{2N-1} i((n-k)T) \cdot z(kT)]}{[R_s + z(0) \cdot T]} \qquad (7)$$

However, this approach may not be optimum, since the impedance magnitude can range widely, and could indeed range from minus- to plus-infinity in particular cases. An unattractively-large number of frequency samples would therefore be required to achieve accurate results throughout the frequency band.

We have found that, in general, system function formulations based on the scattering matrix are usually much preferable, since the parameters remain bounded in spite of impedance singularities. For example, let $S_{11}(\omega)$ denote the input voltage reflection coefficient (referred to $R_0$) in Fig. 3, and $h_{11}(kT)$ represent the corresponding impulse response samples. Using a similar approach to that given above, it is straightforward to show that the current may now be calculated from (8), shown at the bottom of the page.

Although these equations may suggest that the time spacing of the output samples must coincide with the impulse response spacing, in fact this is not so: a simple modification to the convolution expression allows any time step to be used which

$$i(nT) = \left[\frac{\{1 - h_{11}(0) \cdot T\} \cdot e(nT) - T \cdot \sum_{k=1}^{2N-1} C((n-k)T) \cdot h_{11}(kT)}{R_0 \cdot \{1 + h_{11}(0) \cdot T\} + R_s \cdot \{1 - h_{11}(0) \cdot T\}}\right]$$

where:

$$C(mT) = [e(mT) + \{R_0 - R_s\} \cdot i(mT)] \qquad (8)$$

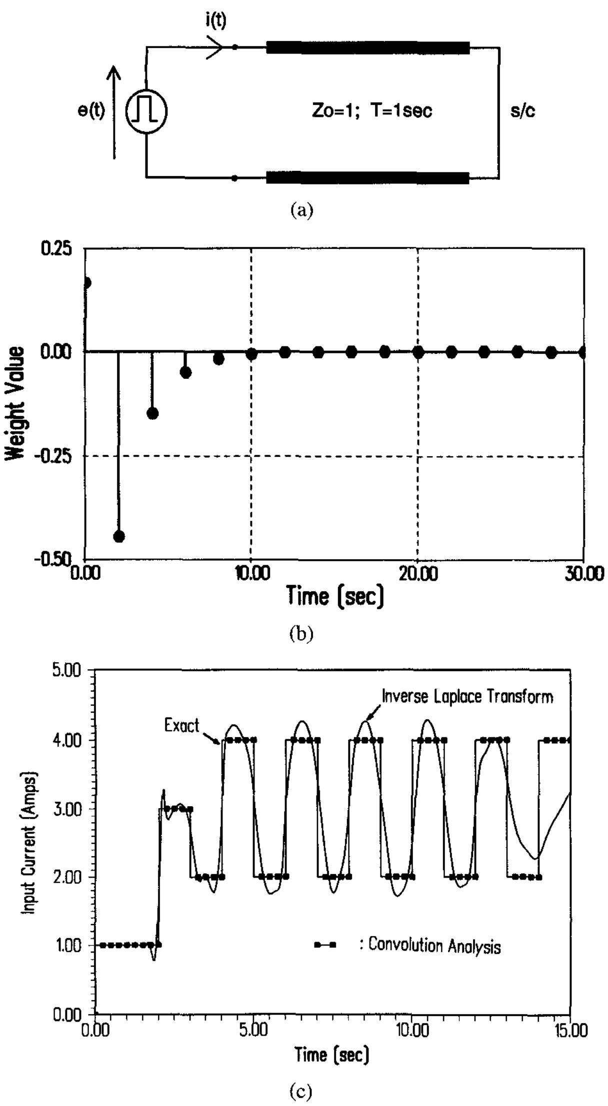


Fig. 4. (a) Transient response of short-circuited transmission line; (b) impulse response computed from $S_{11}(f)\,[R_0 = 2.Z_0]$; (c) pulse response of input current $\imath(t)$.

is an integer times smaller than this spacing. Varying time-steps may be accommodated by using a suitable interpolation technique. Hence, essentially-arbitrary resolution is possible in the time-domain solution.

There is an interesting alternative view of the above formulations, from the point of view of digital filter concepts. For example, if $R_0 = R_s$, (8) involves a direct calculation of the output (current) as a weighted combination of the present input voltage and past values delayed. The linear system is thus being modelled as a Finite Impulse Response (FIR) digital filter. However, if $R_0 \neq R_s$, the present current sample also depends on past values of the current, and therefore the representation is that of an Infinite Impulse Response (IIR) filter, a situation which also obtains in (7). It may be noted that some important fundamental differences exist between these two types of filter structure [2], [12].

As a simple example of the application of the CC-method, we consider the circuit of Fig. 4(a), where the transient input current is required in response to 1-V input pulse of 3 sec. duration. This example was originally described in [9], where an approach is presented based on numerical inversion of the Laplace Transform. As pointed out in [9], it is extremely difficult to obtain accurate numerical results using the DFT in this case, due to the non-bandlimited character of the input signal, although the exact result is, in fact, readily inferred by inspection.

Using a scattering parameter description within the CC-method, if the reference impedance is chosen so that $R_0 = Z_0$, the impulse response has only one non-zero value. However in order to indicate a slightly more general situation, Fig. 4(b) shows the 16-point impulse response calculated with $R_0 = 2.Z_0$, using 9 frequency-domain samples. Fig. 4(c) presents (i) the exact result for the transient input current, (ii) the result using the inverse Laplace Transform method of [9], and (iii) the result using the CC approach. It is seen that the latter provides almost perfect agreement with the exact solution, and it should be emphasised that this agreement continues indefinitely in time.

As a second example, we discuss the low-pass filter circuit in Fig. 5(a). For the present, all transmission lines are assumed ideal, and Fig. 5(b) shows 64-point impulse response representations for $S_{11}$ and $S_{21}$, respectively. As an indication of the frequency-domain interpolation properties of these samples, (i.e. between the 33 frequency points used to generate them), Fig. 5(c) shows the exact magnitude and phase of $S_{21}$, respectively, compared with the CC-interpolated curves, and the agreement is seen to be very good. Fig. 6 presents the transmission voltage response of this structure to a unit voltage-step at the generator. The output voltage is calculated from:

$$v_2(nT) = \frac{1}{2} \cdot \sqrt{\frac{R_{02}}{R_{01}}} \cdot \left\{ \sum_{k=0}^{2N-1} e((n-k)T) \cdot h_{21}(kT) \cdot T \right\} \tag{9}$$

where $h_{21}(kT)$ are impulse response values corresponding to $S_{21}$, with $R_{01}$ and $R_{02}$ being the $S$-parameter reference impedances, coinciding with the impedances actually used in the circuit. Also shown in Fig. 6 is the transient output voltage computed using SPICE, and the agreement is observed to be excellent.

## IV. Non-Periodic System Functions

Consider now a system function which is not inherently periodic (such as that depicted with the block symbols in Fig. 2), and let us assume in the following that some process is available to generate a value for the system function at any specified frequency. Use of the causal-convolution method in such cases requires a preliminary knowledge of the distribution of spectral energy of the excitation signal, so that a frequency $\omega_m$ may be specified, beyond which the energy content of the signal is relatively small. This frequency is referred to as the *boundary frequency* in the following. In essence, one then forms the periodic extension of the system function outside this frequency range, and the CC-technique is applied as in the previous section. Note that it is not required that the excitation signal be strictly band-limited, merely that the

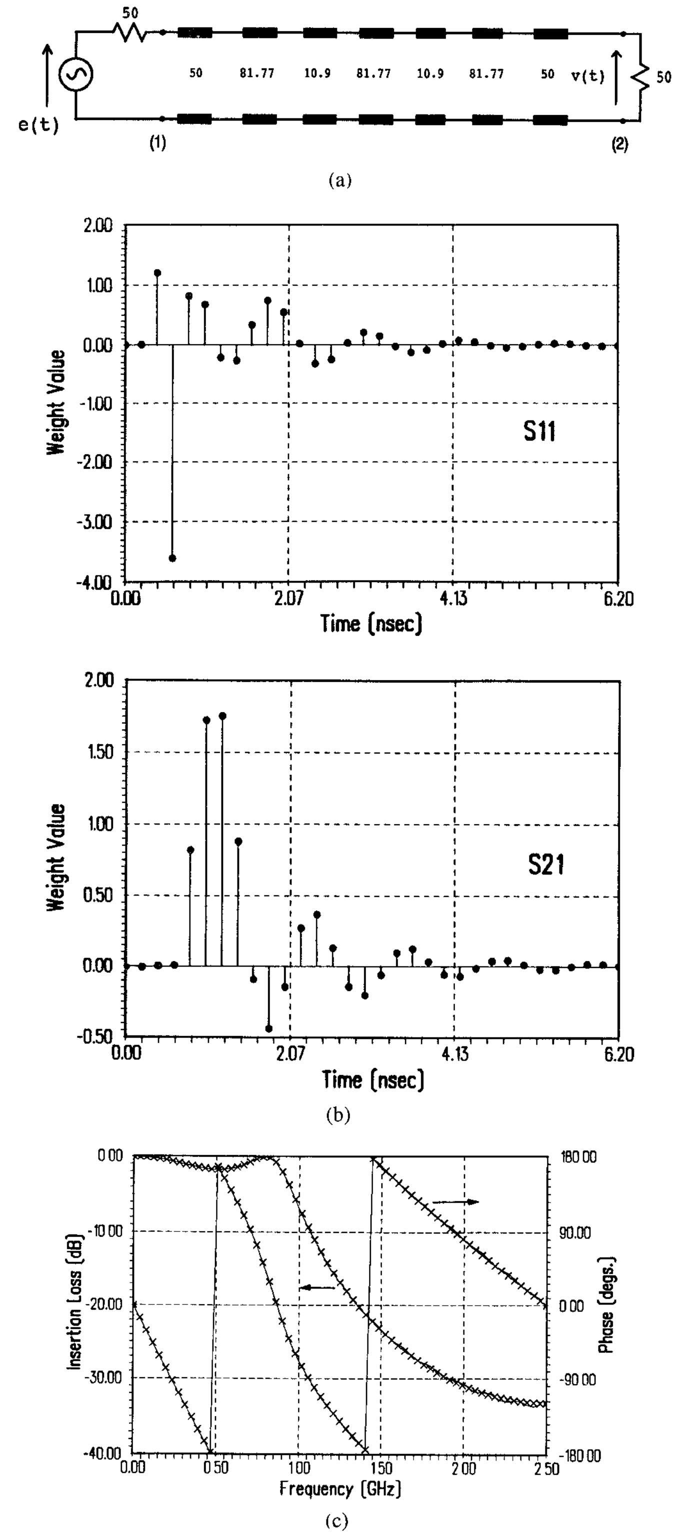


Fig. 5. (a) Idealized low-pass filter structure (lines = 90 degs. at 2.5 GHz); (b) impulse response tunctions calculated from $S_{11}(f)$ and $S_{21}(f)$ ; (c) frequency-domain interpolation properties of $S_{21}(f)$ impulse response (x x: Exact; ______ : Interpolated)

spectral amplitude decreases sufficiently at high frequencies. The worst case situation in practice will arise in connection with step functions, for which the amplitude falls off as $(1/\omega)$, but even in this case it is usually not necessary to go to very high frequencies, in order to obtain accurate results.

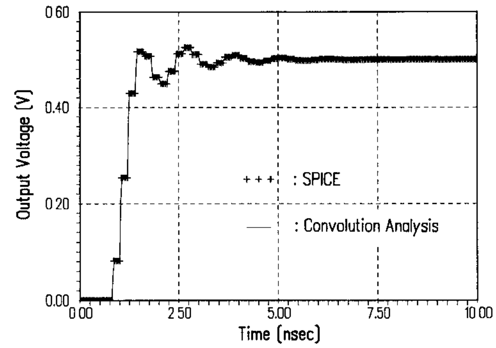


Fig. 6. Output voltage response of low-pass filter structure to unit-step input; SPICE and CC-method compared.

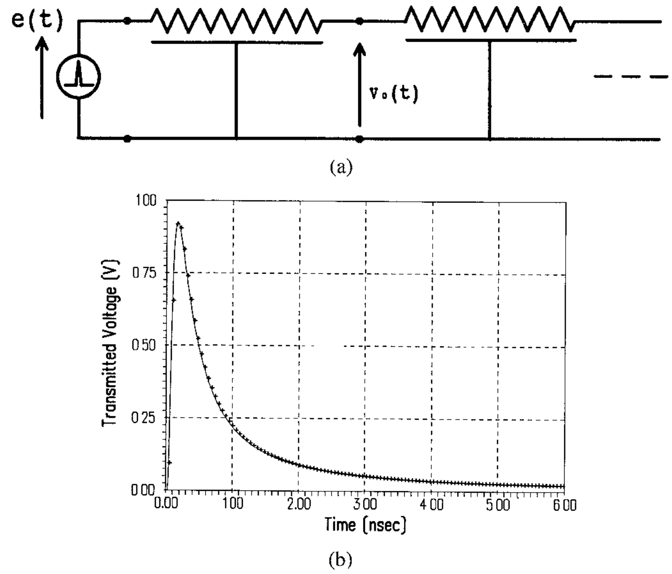


Fig. 7. (a) RC-lne structure; (b) impulse response $V0(t)$ at $1 = 100$ mm: ++ CC-method; ______ analytic ($r = 0.25$ ohm/mm; $c = 0.4pF$/mm).

Nevertheless, it is necessary, at least in principle, that the operation of periodically-extending the system function should not:

1) introduce any discontinuities in the real or imaginary parts in the vicinity of the boundary frequency;
2) violate the assumption of causality at all frequencies.

In practice, it is often very difficult to meet both requirements simultaneously, although it is possible to take measures which greatly minimise the difficulties, without significantly compromising accuracy. In experimenting with these issues, a number of effective strategies have been developed, as outlined in the following:

### *A. System Functions Which Become Very Small at High Frequencies*

As an example of this situation, we may consider the transient response of an RC-line, as shown in Fig. 7(a). The line is assumed to be semi-infinitely long, a delta-function (impulse) voltage is applied at the input, and the voltage response is sought at a specified distance from the generator.

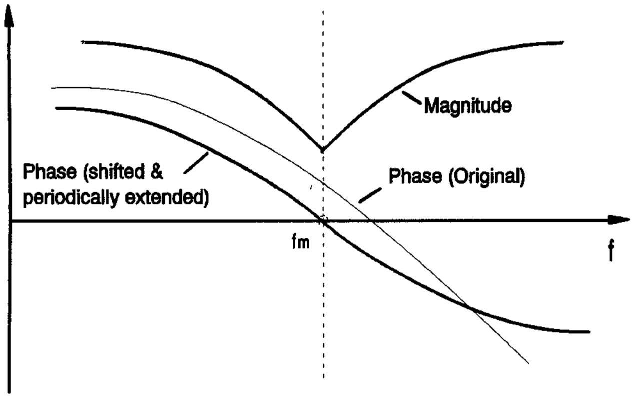


Fig. 8. Effect of phase-shift and periodic extension on magnitude and phase system functions.

In this case, an analytical inversion of the Laplace Transform becomes possible [13], and the transient response is:

$$v_0(t) = \mathcal{L}^{-1}\{e^{-\sqrt{rcs}\cdot l}\} = \frac{\sqrt{rc}\cdot l}{2\sqrt{\pi}\cdot t^{3/2}} \cdot \exp\left[-\frac{rc\cdot l^2}{4\cdot t}\right] \quad (10)$$

Because $S_{21}$ quickly becomes small as the frequency is increased, the CC method may be used directly based on the frequency-domain representation of the RC-line, and the above-mentioned difficulties have negligible effect. Fig. 7(b) compares the transient response calculated in this way with the analytical response, and the agreement is seen to be excellent. It must be stressed that this analysis is based strictly on frequency-domain data: there is no use of lumped approximations etc. In fact, the CC method seems to be very well suited to the efficient transient analysis of general RC-type structures.

### *B. Conversion to an All-Transmission Line Network*

Microwave passive circuit models often consist of a mixture of transmission lines and lumped reactive elements, with the latter modelling discontinuity effects, package parasitics etc. as well as discrete passive components (e.g. bias-block capacitors). In reality, however, a circuit element for which a "lumped" model is adequate at say 10 GHz, is likely to show strongly-distributed behaviour at 100 GHz. This fact can be used to advantage to produce a periodic system function for the CC-method if a relatively large boundary frequency is used, or is made necessary by the spectrum of the excitation signal. The method, related to the standard Richard's transformation of network theory, involves transforming a given circuit to an all-transmission line equivalent, with the electrical length of each line being expressed as a multiple of 90° at the boundary frequency. For example, a series inductor may be converted to a series short-circuited stub, one quarter-wavelength long at $\omega_m$; a capacitor becomes an open-circuited stub etc. The stub impedances may be chosen to make the representations exact at some chosen frequency within the band. The resulting circuit will produce naturally-periodic system functions (with ideal lines), and will only become a poor approximation to the original circuit close to the (large) boundary frequency. But at this frequency, the accuracy of the circuit model is probably suspect, for the physical reasons given above, and there is, by definition, only a small amount of excitation signal spectral energy in any case.

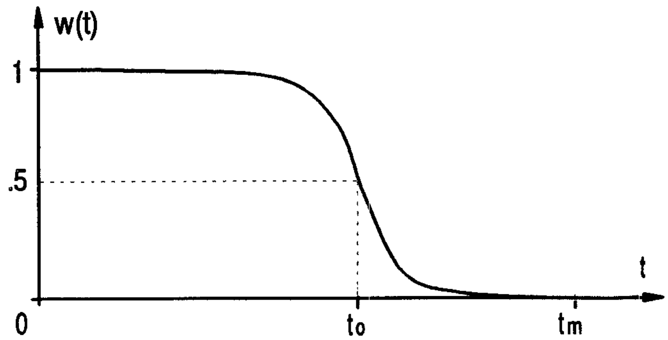


Fig. 9. General shape of one-sided window function applied to impulse response waveform in range $[0, t_m]$.

### *C. Conversion to Non-Minimum-Phase System Function*

Probably the most effective technique for avoiding discontinuities at the boundary frequency is to apply a linear phase shift to the system function, so that the imaginary part becomes zero at this frequency. In fact, this is a necessary requirement for an application of the FFT algorithm to the evaluation of the truncated version of (5). Following derivation of the impulse-response samples, the system function to which they interpolate is of course, no longer minimum-phase, but the effect of the extra phase shift can easily be removed, if desired, by a simple shift operation in the time domain. This technique gives especially good results if there is some latitude in the choice of a boundary frequency value, so that a value may be chosen at which the system magnitude function passes through a natural stationary point. The reason for this is illustrated in Fig. 8, where it is seen that although a suitable phase shift will produce an almostcontinuous, periodic system phase function for an arbitrary choice of $f_m$, in general, the magnitude part of the resulting function will have discontinuities in its higher-order derivatives at this frequency. Examples of this general process are given in the next section, but when used properly, the effect of the shift technique on reducing discontinuity effects can be highly beneficial.

### *D. Use of Time-Domain Windowing*

Window functions are widely used in digital signal processing, especially the symmetrical time-domain windows used in FIR-filter design (Raised-Cosine, Hamming, Kaiser etc [2]). One-sided window functions of the general shape indicated in Fig. 9, can also be useful within the present CC technique, especially for the suppression of residual non-causality arising from the periodic-extension operation on general system functions. Numerous suitable window functions may be envisaged, for example, with reference to Fig. 9:

$$w_{t\leq t_0}(t) = 1 - \frac{1}{2}\cdot\left[\frac{t}{t_0}\right]^{\left(\frac{t_0\ \gamma}{t_m - t_0}\right)}$$
$$w_{t>t_0}(t) = \frac{1}{2}\cdot\left[\frac{t_m - t}{t_m - t_0}\right]^{\gamma} \quad (11)$$

The use of window functions may be illustrated in the context of transient analysis of circuits incorporating lossy and/or dispersive transmission lines, where causal-convolution should

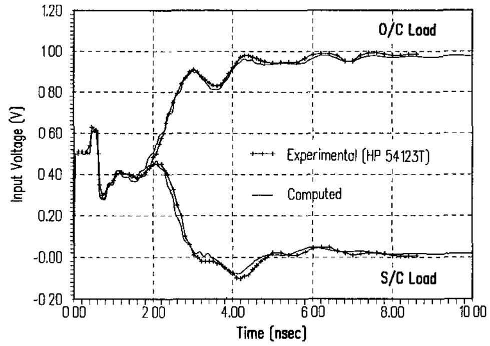


Fig. 10. Computed and measured input voltage step-responses for practical low-pass filter structure, with open-circuit and short-circuit terminations.

have a strong advantage compared to conventional time-domain transient simulation techniques. However, while ideal transmission lines are perfectly periodic in the frequency domain, the inclusion of conductor loss via a $\sqrt{f}$ dependence, for example, or microstrip dispersion effects using a Getsinger-type expression [14], will lead to problems with non-causality when it is attempted simply to periodically-extend a chosen low-frequency range of the system function. In practical application of the CC-method, the non-causality effects show up as a small but perceptible "tail" on the discrete impulse response, which slightly degrades the interpolation performance in the frequency domain. The degradation shows up as a small ripple effect super-imposed on the broad trend of the frequency domain functions. Application of a window function is effective in suppressing this effect, and allows the production of a smooth interpolated system function, although the function no longer passes exactly through the sample values chosen to create the CC impulse response. Extensive experimentation, not reported here, has shown that time-domain windowing appears be a useful technique where smooth, interpolated frequency-domain behaviour is desirable, in cases such as that just described.

## V. Applications and Validation

Several examples of applications of the CC technique have already been given. In this section, we provide a few additional examples, emphasising validation of the technique and discussing its advantages and limitations.

### *A. Lossy, Dispersive Low Pass Filter: Comparison with Experiment*

Earlier on, we discussed the ideal transmission line structure of Fig. 5(a), and showed in Fig. 6 that excellent agreement could be obtained between a SPICE2 transient analysis and a CC analysis for this structure. It is worth mentioning the CC-technique is substantially faster in terms of computer time in this analysis, and this advantage is particularly striking when lines of greatly differing electrical lengths are solved, because SPICE becomes very inefficient in such cases. A practical version of the structure of Fig. 5(a) was realised on conventional PCB board, a substrate deliberately used so as to accentuate loss and other non-idealities at microwave frequencies. The phase-shift technique described in the previous section could be used to obtain 64-point impulse responses corresponding to $S_{11}(f)$ and $S_{21}(f)$ responses, again with excellent interpolation properties. Fig. 10 shows the computed and measured input voltage step-responses for the structure, for both an open-circuit and a short-circuit termination at the output. The measured data was obtained from a HP 54123T high-speed oscilloscope system, and the agreement between computed and measured results is observed to be very good. The effects of loss and dispersion are very apparent in comparison with the idealised result of Fig. 6, and of course this kind of transient numerical analysis would not be possible using SPICE2.

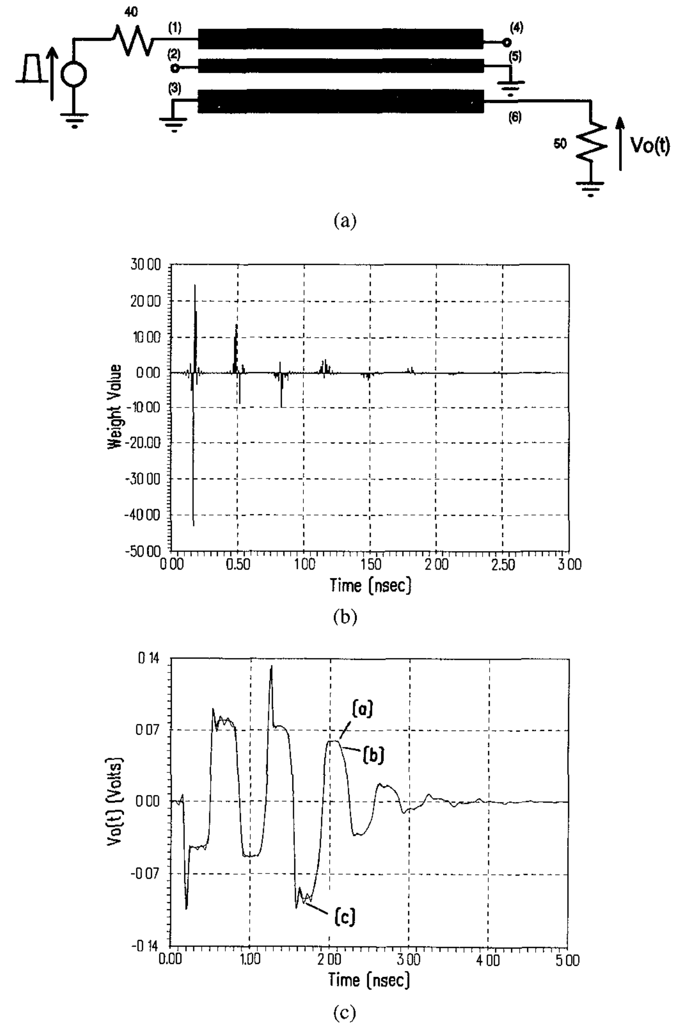


Fig. 11. (a) Triple asymmetric coupled microstrip line structure. Transient output $V_0^{(t)}$ required for pulse voltage input. Widths and spacings (from top): $W1$ = 16 mil; $S1$ = 8 mil; $W2$ =8 mil; $S2$ = 10 mil; $W3$ = 20 mil; Length = 800 mil, realized on 25-mil alumina substrate. Loss and dispersion included. (b) Impulse response of triple-coupled microstrip line structure, corresponding to $S_{16}(f)$. (c) Transient output voltage $V_0(t)$ computed using CC-method with various boundary frequencies: (a) 50 GHz; (b) 36 GHz; (c) 10 GHz.

### *B. Asymmetrical Triple Coupled Line Structure*

A further example is shown in Fig. 11(a), where three microstrip lines are coupled with unequal widths and spacings. The frequency domain data in this case is obtained from a commercial electromagnetic-analysis software product, and the $S_{16}$ response, for example, may be processed to produce the impulse response shown in Fig. 11(b), based on 257 frequency domain points up to about 50 GHz. Assuming now a trapezoidal-type pulse input of 1.1 nS duration with equal rise and fall times of 50 pS, Causal-Convolution analysis produces time-domain voltage responses at Port (6) as shown in Fig. 10(c). In the latter diagram, the waveforms displayed as (a) and (b), respectively, show the effect of a move of the boundary frequency from its original value down to, say, 36 GHz. Because there is in fact relatively little pulse spectral energy above 36 GHz, the effect on the time-domain response is virtually imperceptible. However, if the boundary frequency is reduced to only 10 GHz, waveform (c) indicates that some changes are becoming apparent in the response, and this choice of frequency is evidently too low. Experience to date suggests that if the boundary frequency is at least of the order of the reciprocal of the shortest rise-time of the excitation (20 GHz in the case just considered), application of the CC-technique will produce very acceptable results in any transient analysis.

## VI. Conclusion

A new method has been presented for the transient analysis of causal linear systems described in the frequency domain. The central novelty lies in the method of determining the impulse response, which is interpreted as a truly discrete function corresponding to a periodically-extended system function. The impulse response may be computed with high numerical efficiency, while retaining excellent interpolation properties with respect to the original system function. Convolution operations are also intrinsically in the form of a sum-of-products calculation. The technique is capable of handling arbitrary excitation signals, unlike DFT-based analysis, for example, which requires bandlimited excitations. This causal-convolution technique produces highly accurate results provided the boundary frequency beyond which the original system function becomes periodically-extended, is such as to contain most of the spectral energy in the excitation signal.

The operation of periodic-extension on a given causal complex-valued system function, must be performed with some care. However, provided some simple guidelines are followed as indicated in this paper, very successful results may be obtained in quite general cases. Indeed, we have used the technique successfully on frequency-domain data from electromagnetic simulators, network analyser measurement data etc. We have had no experience to date of any difficulty from numerical overflow or instability, and accurate transient analysis is invariably possible with great efficiency.

While the CC-technique has been shown to be of considerable value in linear-circuit transient analysis, its real power is apparent in nonlinear applications, where it provides an extremely simple interface between nonlinear and linear parts of a system, with the non-linear parts analysed in the time-domain, and the linear parts in the frequency domain. We have successfully demonstrated this extension, and intend to report on this work in the near future.

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**Thomas J. Brazil** (M'87) graduated from University College Dublin in 1973 with a First Class Honours Degree in electrical engineering. He was awarded the Ph.D. degree in 1977 by the National University of Ireland for research in electronic engineering, the title of his dissertation being "Computer Simulation of IMPATT Diodes."

He was subsequently employed as a senior scientist working on microwave sub-system development at Plessey Research (Caswell) UK from 1977 to 1979. After spending a year as a lecturer in the Department of Electronic Engineering, University of Birmingham, UK, he returned as a lecturer to UCD in 1980, and he is now a Professor in the Department of Electronic and Electrical Engineering, and head of the Microwave Research Laboratory in UCD. His research interests are in the general area of non-linear microwave computer-aided design. These include the development of algorithms for the efficient CAD of mixed time-domain and frequency-domain non-linear systems, as well as work on device large-signal modelling and characterization techniques, with particular emphasis on applications to microwave transistor devices such as the GaAs FET, HEMT, BJT, and the HBT. He also has interests in optical device modelling and topics in passive circuit design (e.g. lossy multiple lines and CPW), and non-linear microwave sub-system design and development.